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## $\alpha$ Ig-NORMAL AND $\alpha$ Ig-REGULAR SPACES IN IDEAL TOPOLOGICAL SPACE

S.Margathavalli \*  
D.Vinodhini\*\*

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### Abstract

In this paper,  $\alpha$ Ig-normal spaces,  $\alpha$ Ig\*-normal spaces,  $\alpha$ Ig-regular spaces and mildly  $\alpha$ Ig-normal spaces are introduced. Characterizations and properties of such new notions are studied. Some preservation theorems for these normal spaces are obtained.

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### Keywords:

$\alpha$ Ig-closed sets;  
 $\alpha$ Ig-normal spaces;  
 $\alpha$ Ig\*-normal spaces;  
 $\alpha$ Ig-regular spaces;  
mildly  $\alpha$ Ig-normal spaces.

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### Author correspondence:

D.Vinodhini,  
Department of Mathematics,  
SVS College of Engineering, Coimbatore, Tamilnadu, India

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## 1. Introduction

An Ideal  $I$  on a topological space  $(X, \tau, I)$  is defined as a non-empty collection  $I$  of subsets of  $X$  satisfying the following two conditions (i) if  $A \in I$  and  $B \subset A$ , then  $B \in I$  (ii) If  $A \in I$  and  $B \in I$ , then  $A \cup B \in I$ . We will make use of the basic facts about the local functions without mentioning it explicitly. A Kuratowski closure operator  $cl^*(.)$  for a topology  $\tau^*(\tau, I)$  called the  $*$ -topology, finer than  $\tau$  is defined by  $cl^*(A) = A \cup A^*(\tau, I)$ . When there is no chance for confusion, we simply write  $A^*$  instead of  $A^*(\tau, I)$  and  $\tau^*$  for  $\tau^*(\tau, I)$ . For a subset  $A \subset X$ ,  $A^*(\tau, I) = \{x \in X / U \cap A \notin I \text{ for each neighborhood } U \text{ of } x\}$ . For every ideal topological space  $(X, \tau, I)$ , there exists a topology generated by  $\beta(I, \tau) = \{U - J / U \in \tau \text{ and } J \in I\}$ . In general  $\beta(I, \tau)$  is not always a topology [5]. If  $I$  is an ideal on  $X$ , then it is called ideal space. By an ideal space, we always mean an ideal topological space with no separation properties assumed.

The notions of the normal spaces in ideal topological spaces are highly developed and used extensively in many practical and engineering problems, computational topology

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\* Doctorate Program, Linguistics Program Studies, Udayana University Denpasar, Bali-Indonesia (9 pt)

\*\* STIMIK STIKOM-Bali, Renon, Denpasar, Bali-Indonesia

for geometric design, computer-aided geometric design, engineering design research and mathematical sciences. Hence we studied new types of normal spaces and regular spaces and obtained some of their properties in ideal topological spaces. Navaneethakrishnan M, Paulraj Joseph J and Sivaraj D [8] were introduced Ig-normal spaces and Ig-regular spaces in ideal topological spaces. Ravia O, Rajasekaranb I, Vijayac S, Murugesand S [11] were introduced mildly \*-normal spaces in ideal topological spaces. In this chapter, we introduce the concept of  $\alpha$ Ig-normal spaces,  $\alpha$ Ig\*-normal spaces,  $\alpha$ Ig-regular spaces and mildly  $\alpha$ Ig-normal spaces and discuss about their properties.

## 2. Preliminaries

**Definition 1.1:[11]** Let  $(X, \tau)$  be a topological space. A subset  $A$  of  $X$  is called

- (i) regular open if  $A = \text{int}(\text{cl}(A))$ .
- (ii) regular closed if  $A = \text{cl}(\text{int}(A))$ .

The complement of regular open set is regular closed.

The collection of regular open (resp. regular closed) subsets of  $X$  is denoted by  $\text{RO}(X)$  (resp.  $\text{RC}(X)$ ).

**Remark 1.2:[11]** In any topological spaces, the following holds.

Every regular closed set is a closed set.

**Definition 1.3:[11]** Let  $(X, \tau, I)$  be an ideal topological space. Then  $I$  is said to be completely codense [3] if  $\text{PO}(X) \cap I = \{\emptyset\}$ , where  $\text{PO}(X)$  is the family of all preopen sets in  $(X, \tau)$ .

**Lemma 1.4:[2]** Let  $(X, \tau, I)$  be an ideal topological space and  $A \subseteq X$ . If  $A \subseteq A^*$ , then  $A^* = \text{cl}(A^*) = \text{cl}(A) = \text{cl}^*(A)$ .

**Definition 1.5:[8]** A subset  $A$  of an ideal topological space  $(X, \tau, I)$  is Ig-closed [14], if  $A^* \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open.

The following Lemmas, Result, Definitions, Remarks and Theorem will be useful in the sequel.

**Lemma 1.6: [3]** Let  $(X, \tau, I)$  be an ideal topological space and  $A, B$  subsets of  $X$ . Then the following properties hold:

- (i)  $A \subseteq B \Rightarrow A^* \subseteq B^*$ ,
- (ii)  $A^* = \text{cl}(A^*) \subseteq \text{cl}(A)$ ,
- (iii)  $(A^*)^* \subseteq A^*$ ,
- (iv)  $(A \cup B)^* = A^* \cup B^*$ ,
- (v)  $(A \cap B)^* \subseteq A^* \cap B^*$ .

**Definition 1.7:[8]** An ideal topological space  $(X, \tau, I)$  is said to be a normal space, if for every pair of disjoint closed sets  $A$  and  $B$ , there exist disjoint open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$ .

**Definition 1.8: [8]** An ideal topological space  $(X, \tau, I)$  is said to be a Ig-normal space, if for every pair of disjoint closed sets  $A$  and  $B$ , there exist disjoint Ig-open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$ .

**Definition 1.9:[8]** An ideal topological space  $(X, \tau, I)$  is said to be a  $gI$ -normal space, if for every pair of disjoint  $Ig$ -closed sets  $A$  and  $B$ , there exist disjoint open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$ .

This is a stronger form of normal space.

**Definition 1.10:[8]** An ideal space  $(X, \tau, I)$  is said to be an  $Ig$ -regular space if for each pair consisting of a point  $x$  and a closed set  $B$  not containing  $x$ , there exist disjoint  $Ig$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $B \subseteq V$ .

**Definition 1.11:[2]** An ideal space  $(X, \tau, I)$  is said to be an  $Irwg$ -normal space, if for every pair of disjoint closed sets  $A$  and  $B$ , there exist disjoint  $Irwg$ -open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$ .

**Definition 1.12:[2]** An ideal space  $(X, \tau, I)$  is said to be an  $Irwg$ -regular space if for each pair consisting of a point  $x$  and a closed set  $B$  not containing  $x$ , there exist disjoint non empty  $Irwg$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $B \subseteq V$ .

**Definition 1.13: [11]** An ideal topological space  $(X, \tau, I)$  is called  $*$ -normal if for any pair of disjoint closed sets  $A$  and  $B$  of  $X$ , there exist disjoint  $*$ -open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$ .

**Definition 1.14: [11]** An ideal topological space  $(X, \tau, I)$  is called mildly  $*$ -normal if for every pair of disjoint  $H, K \in RC(X)$ , there exist disjoint  $*$ -open sets  $U$  and  $V$  such that  $H \subseteq U$  and  $K \subseteq V$ .

### 3. $\alpha Ig$ -Normal and $\alpha Ig$ -Regular Spaces

In this section, we introduce  $\alpha Ig$ -normal space,  $\alpha Ig^*$ -normal space and  $\alpha Ig$ -regular spaces and study their relations with existing ones.

**Definition 3.1:** An ideal space  $(X, \tau, I)$  is said to be an  $\alpha Ig$ -normal space, if for every pair of disjoint closed sets  $A$  and  $B$ , there exist disjoint  $\alpha Ig$ -open sets  $U$  and  $V$  in  $X$  such that  $A \subseteq U$  and  $B \subseteq V$ .

**Definition 3.2:** An ideal space  $(X, \tau, I)$  is said to be an  $\alpha Ig^*$ -normal space, if for every pair of disjoint  $\alpha Ig$ -closed sets  $A$  and  $B$ , there exist disjoint open sets  $U$  and  $V$  in  $X$  such that  $A \subseteq U$  and  $B \subseteq V$ .

This is a stronger form of normal spaces. We characterize  $\alpha Ig^*$ -normal spaces and give various properties of such spaces.

**Definition 3.3:** An ideal space  $(X, \tau, I)$  is said to be an  $\alpha Ig$ -regular space, if for each pair consisting of a point  $x$  and a closed set  $B$  not containing  $x$ , there exist disjoint  $\alpha Ig$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $B \subseteq V$ .

Various characterizations of  $\alpha Ig$ -regular space are also given. The following example shows that an  $\alpha Ig$ -normal space is not a normal space.

**Example 3.4:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\Phi, \{b\}, \{a, b\}, X\}$  and  $I = \{\Phi, \{b\}\}$ . Here, every open set is  $*$ -closed and so, every subset of  $X$  is  $\alpha Ig$ -closed and hence every subset of  $X$  is  $\alpha Ig$ -open. This implies that  $(X, \tau, I)$  is  $\alpha Ig$ -normal,  $\{a\}$  and  $\{c\}$  are disjoint closed subsets of  $X$  which are not separated by disjoint open sets and so  $(X, \tau)$  is normal.

The following theorem gives characterizations of  $\alpha Ig$ -normal spaces.

**Theorem 3.5:** Let  $(X, \tau, I)$  be an ideal space. Then the following are equivalent.

- (i)  $X$  is  $\alpha Ig$ -normal.

(ii) For every pair of disjoint closed sets  $A$  and  $B$ , there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $A \subset U$  and  $B \subset V$ .

(iii) For every closed set  $A$  and an open set  $V$  containing  $A$ , there exists an  $\alpha$ Ig-open set  $U$  such that  $A \subset U \subset \text{cl}^*(U) \subset V$ .

**Proof:** (i)  $\Rightarrow$  (ii): The proof follows from the definition of  $\alpha$ Ig-normal spaces

(ii)  $\Rightarrow$  (iii): Let  $A$  be a closed set and  $V$  be an open set containing  $A$ . Since  $A$  and  $X - V$  are disjoint closed sets, there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $W$  such that  $A \subset U$  and  $X - V \subset W$ . Again  $U \cap W = \Phi$  implies that  $U \cap \text{int}^*(W) = \Phi$ . Therefore  $U \subset X - \text{int}^*(W)$  and so  $\text{cl}^*(U) \subset X - \text{int}^*(W)$ . Again  $X - V \subset W$  implies that  $X - V \subset \text{int}^*(W)$  [[3]Theorem 3.31] and so  $X - \text{int}^*(W) \subset V$ . Thus, we have  $A \subset U \subset \text{cl}^*(U) \subset X - \text{int}^*(W) \subset V$  which proves (iii).

(iii)  $\Rightarrow$  (i): Let  $A$  and  $B$  be two disjoint closed subsets of  $X$ . By hypothesis, there exists an  $\alpha$ Ig-open set  $U$  such that  $A \subset U \subset \text{cl}^*(U) \subset X - B$ . If  $W = X - \text{cl}^*(U)$ , then  $U$  and  $W$  are the required disjoint  $\alpha$ Ig-open sets containing  $A$  and  $B$  respectively. So,  $(X, \tau, I)$  is  $\alpha$ Ig-normal.

**Theorem 3.6:** Let  $(X, \tau, I)$  be an  $\alpha$ Ig-normal space. If  $F$  is closed and  $A$  is a  $\alpha$ -closed set such that  $A \cap F = \Phi$ , then there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $A \subset U$  and  $F \subset V$ .

**Proof:** Since  $A \cap F = \Phi$ ,  $A \subset X - F$  where  $X - F$  is open. Therefore, by hypothesis,  $\text{cl}(A) \subset X - F$ . Since  $\text{cl}(A) \cap F = \Phi$  and  $X$  is  $\alpha$ Ig-normal, there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $\text{cl}(A) \subset U$  and  $F \subset V$ . Hence the proof.

**Theorem 3.7:** Let  $(X, \tau, I)$  be an ideal space which is  $\alpha$ Ig-normal. Then the following hold:

(i) For every closed set  $A$  and every  $\alpha$ -open set  $B$  containing  $A$ , there exists an  $\alpha$ Ig-open set  $U$  such that  $A \subset \text{int}^*(U) \subset U \subset B$ .

(ii) For every  $\alpha$ -closed set  $A$  and every open set  $B$  containing  $A$ , there exists an  $\alpha$ Ig-closed set  $U$  such that  $A \subset U \subset \text{cl}^*(U) \subset B$ .

**Proof:** (i) Let  $A$  be a closed set and  $B$  be a  $\alpha$ -open set containing  $A$ . Then  $A \cap (X - B) = \Phi$ , where  $A$  is closed and  $X - B$  is  $\alpha$ -closed. By Theorem 3.6, there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $A \subset U$  and  $X - B \subset V$ . Since  $U \cap V = \Phi$ , we have  $U \subset X - V$  and  $A \subset \text{int}^*(U)$ . Therefore,  $A \subset \text{int}^*(U) \subset U \subset X - V \subset B$ . This proves (i).

(ii) Let  $A$  be  $\alpha$ -closed and  $B$  be an open set containing  $A$ . Then  $X - B$  is a closed set contained in the  $\alpha$ -open set  $X - A$ . By (i), there exists an  $\alpha$ Ig-open set  $V$  such that  $X - B \subset \text{int}^*(V) \subset V \subset X - A$ . Therefore,  $A \subset X - V \subset \text{cl}^*(X - V) \subset B$ . If  $U = X - V$ , then  $A \subset U \subset \text{cl}^*(U) \subset B$  and so  $U$  is the required  $\alpha$ Ig-closed set. This proves (ii).

**Theorem 3.8:** Every  $\alpha$ Ig\*-normal space is a normal space.

**Proof:** An ideal space  $(X, \tau, I)$  is said to be an  $\alpha$ Ig\*-normal space, if for each pair of disjoint  $\alpha$ Ig-closed sets  $A$  and  $B$ , there exist disjoint open sets  $U$  and  $V$  in  $X$  such that  $A \subset U$  and  $B \subset V$ . Since every closed set is  $\alpha$ Ig-closed, every  $\alpha$ Ig\*-normal space is normal.

**Remark 3.9:** The following example shows that a normal space need not be an  $\alpha$ Ig\*-normal space.

**Example 3.10:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\Phi, \{a\}, X\}$  and  $I = \{\Phi, \{a\}\}$ . Now,  $A = \{a\}$  and  $B = \{b, c, d\}$  are disjoint  $\alpha$ Ig-closed sets, but they are not separated by disjoint open sets. So,  $(X, \tau, I)$  is not  $\alpha$ Ig\*-normal space. Since there is no pair of disjoint closed sets,  $(X, \tau, I)$  is normal.

Theorem 3.11 and 3.12 below give characterizations of  $\alpha$ Ig\*-normal spaces.

**Theorem 3.11:** In an ideal space  $(X, \tau, I)$ , the following are equivalent.

- (i)  $X$  is  $\alpha$ Ig\*-normal.
- (ii) For every  $\alpha$ Ig-closed set  $A$  and every  $\alpha$ Ig-open set  $B$  containing  $A$ , there exists an open set  $U$  of  $X$  such that  $A \subset U \subset \text{cl}(U) \subset B$ .

**Proof:** (i)  $\Rightarrow$  (ii): Let  $A$  be an  $\alpha$ Ig-closed set and  $B$  be an  $\alpha$ Ig-open set containing  $A$ . Since  $A$  and  $X - B$  are disjoint  $\alpha$ Ig-closed sets, there exists disjoint open sets  $U$  and  $V$  such that  $A \subset U$  and  $X - B \subset V$ . Now  $U \cap V = \Phi$  implies that  $\text{cl}(U) \subset X - V$ . Therefore,  $A \subset U \subset \text{cl}(U) \subset X - V \subset B$ . This proves (i).

(ii)  $\Rightarrow$  (i): Suppose  $A$  and  $B$  are disjoint  $\alpha$ Ig-closed sets, then the  $\alpha$ Ig-closed set  $A$  is contained in the  $\alpha$ Ig-open set  $X - B$ . By hypothesis, there exists an open set  $U$  of  $X$  such that  $A \subset U \subset \text{cl}(U) \subset X - B$ . If  $V = X - \text{cl}(U)$ , then  $U$  and  $V$  are disjoint open sets containing  $A$  and  $B$  respectively. Therefore,  $(X, \tau, I)$  is  $\alpha$ Ig\*-normal space.

**Theorem 3.12:** In an ideal space  $(X, \tau, I)$ , the following are equivalent.

- (i)  $X$  is  $\alpha$ Ig\*-normal.
- (ii) For each pair of disjoint  $\alpha$ Ig-closed subsets  $A$  and  $B$  of  $X$ , there exists an open set  $U$  of  $X$  containing  $A$  such that  $\text{cl}(U) \cap B = \Phi$ .
- (iii) For each pair of disjoint  $\alpha$ Ig-closed subsets  $A$  and  $B$  of  $X$ , there exists an open set  $U$  containing  $A$  and an open set  $V$  containing  $B$  such that  $\text{cl}(U) \cap \text{cl}(V) = \Phi$ .

**Proof:** (i)  $\Rightarrow$  (ii): Suppose that  $A$  and  $B$  are disjoint  $\alpha$ Ig-closed subsets of  $X$ . Then the  $\alpha$ Ig-closed set  $A$  is contained in the  $\alpha$ Ig-open set  $X - B$ . By Theorem 3.11, there exists an open set  $U$  such that  $A \subset U \subset \text{cl}(U) \subset X - B$ . Therefore,  $U$  is the required open set containing  $A$  such that  $\text{cl}(U) \cap B = \Phi$ .

(ii)  $\Rightarrow$  (iii): Let  $A$  and  $B$  be two disjoint  $\alpha$ Ig-closed subsets of  $X$ . By hypothesis, there exists an open set  $U$  containing  $A$  such that  $\text{cl}(U) \cap B = \Phi$ . Also,  $\text{cl}(U)$  and  $B$  are disjoint  $\alpha$ Ig-closed sets of  $X$ . By hypothesis, there exists an open set  $V$  containing  $B$  such that  $\text{cl}(U) \cap \text{cl}(V) = \Phi$ .

(iii)  $\Rightarrow$  (i): Let  $A$  and  $B$  be two disjoint  $\alpha$ Ig-closed subsets of  $X$ . By hypothesis, there exists an open set  $U$  containing  $A$  and an open set  $V$  containing  $B$ . Therefore,  $X$  is  $\alpha$ Ig\*-normal.

**Theorem 3.13:** Let  $(X, \tau, I)$  be an  $\alpha$ Ig\*-normal space. If  $A$  and  $B$  are disjoint  $\alpha$ Ig-closed subsets of  $X$ , then there exist disjoint open sets  $U$  and  $V$  such that  $\text{cl}^*(A) \subset U$  and  $\text{cl}^*(B) \subset V$ .

**Proof:** Suppose that  $A$  and  $B$  are disjoint  $\alpha$ Ig-closed sets. By Theorem 3.12, there exists an open set  $U$  containing  $A$  and an open set  $V$  containing  $B$  such that  $\text{cl}(U) \cap \text{cl}(V) = \Phi$ . Since  $A$  is  $\alpha$ Ig-closed,  $A \subset U$  implies that  $\text{cl}^*(A) \subset U$ . Similarly,  $B$  is  $\alpha$ Ig-closed,  $B \subset V$  implies that  $\text{cl}^*(B) \subset V$ . Hence the proof.

**Theorem 3.14:** Let  $(X, \tau, I)$  be an  $\alpha$ Ig\*-normal space. If  $A$  is an  $\alpha$ Ig-closed set and  $B$  is an  $\alpha$ Ig-open set containing  $A$ , then there exists an open set  $U$  such that  $A \subset \text{cl}^*(A) \subset U \subset \text{int}^*(B) \subset B$ .

**Proof:** Suppose  $A$  is an  $\alpha$ Ig-closed set and  $B$  is an  $\alpha$ Ig-open set containing  $A$ . Since  $A$  and  $X - B$  are disjoint  $\alpha$ Ig-closed sets, by Theorem 3.13, there exist disjoint open sets  $U$  and  $V$  such that  $\text{cl}^*(A) \subset U$  and  $\text{cl}^*(X - B) \subset V$ . Now,  $X - \text{int}^*(B) = \text{cl}^*(X - B) \subset V$  implies that  $X - V \subset \text{int}^*(B) \subset B$ . This completes the proof.

**Definition 3.15:** An ideal space  $(X, \tau, I)$  is said to be an  $\alpha$ Ig-regular space, if for each pair consisting of a point  $x$  and a closed set  $B$  not containing  $x$ , there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $x \in U$  and  $B \subset V$ .

**Theorem 3.16:** Every regular space is an  $\alpha$ Ig-regular space.

**Proof:** Let  $(X, \tau, I)$  be a regular space. Then for each pair consisting of a point  $x$  and a closed set  $B$  not containing  $x$ , there exist disjoint open sets  $U$  and  $V$  such that  $x \in U$  and  $B \subset V$ . Since every open set is  $\alpha$ Ig-open. Therefore,  $(X, \tau, I)$  is an  $\alpha$ Ig-regular space.

**Remark 3.17:** The following example shows that an  $\alpha$ Ig-regular space need not be regular.

**Example 3.18:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\Phi, \{b\}, \{a, b\}, X\}$  and  $I = \{\Phi, \{b\}\}$ . Here, every open set is  $\alpha$ Ig-closed and so, every subset of  $X$  is  $\alpha$ Ig-closed and hence every subset of  $X$  is  $\alpha$ Ig-open. This implies that  $(X, \tau, I)$  is  $\alpha$ Ig-regular. Now  $\{c\}$  is a closed set not containing  $a \in X$ .  $\{c\}$  and  $a$  are not separated by disjoint open sets. So  $(X, \tau, I)$  is not regular.

The following theorem gives a characterization of  $\alpha$ Ig-regular space.

**Theorem 3.19:** In an ideal space  $(X, \tau, I)$ , the following are equivalent.

- (i)  $X$  is  $\alpha$ Ig-regular.
- (ii) For every closed set  $B$  not containing  $x \in X$ , there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $x \in U$  and  $B \subset V$ .
- (iii) For every open set  $V$  containing  $x \in X$ , there exists an  $\alpha$ Ig-open set  $U$  of  $X$  such that  $x \in U \subset \text{cl}^*(U) \subset V$ .

**Proof:** (i)  $\Rightarrow$  (ii): Let  $X$  is  $\alpha$ Ig-regular. Then by definition, we have for every closed set  $B$  not containing  $x \in X$ , there exists disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $x \in U$  and  $B \subset V$ . Hence (ii)

(ii)  $\Rightarrow$  (iii): Let  $V$  be an open set such that  $x \in V$ . Then  $X - V$  is a closed set not containing  $x$ . Therefore, there exist  $\alpha$ Ig-open sets  $U$  and  $W$  such that  $x \in U$  and  $X - V \subset W$ . Now,  $X - V \subset W$  implies that  $X - V \subset \text{int}^*(W)$  [[3] Theorem 3.31] and so  $X - \text{int}^*(W) \subset V$ . Again,  $U \cap W = \Phi$  implies that  $U \cap \text{int}^*(W) = \Phi$ . Therefore  $U \subset X - \text{int}^*(W)$  and so  $\text{cl}^*(U) \subset X - \text{int}^*(W)$ . Hence,  $x \in U \subset \text{cl}^*(U) \subset X - \text{int}^*(W) \subset V$ . This proves (iii).

(iii)  $\Rightarrow$  (i) : Let  $B$  be a closed set not containing  $x$ . By hypothesis, there exists an  $\alpha$ Ig-open set  $U$  such that  $x \in U \subset \text{cl}^*(U) \subset X - B$ . If  $W = X - \text{cl}^*(U)$ , then  $U$  and  $W$  are disjoint  $\alpha$ Ig-open sets such that  $x \in U$  and  $B \subset W$ . This proves (i).

**Theorem 3.20:** If every open subset of an ideal space  $(X, \tau, I)$  is  $\alpha$ Ig-closed, then  $(X, \tau, I)$  is  $\alpha$ Ig-regular.

**Proof:** Suppose every open subset of  $X$  is  $*$ -closed. Since every  $*$ -closed is  $\alpha$ Ig-closed, every subset of  $X$  is  $\alpha$ Ig-closed and hence every subset of  $X$  is  $\alpha$ Ig-open. If  $B$  is a closed set not containing  $x$ , then  $\{x\}$  and  $B$  are the required disjoint  $\alpha$ Ig-open sets containing  $x$  and  $B$  respectively. Therefore,  $(X, \tau, I)$  is  $\alpha$ Ig-regular.

The following Example 3.21 shows that the reverse direction of the above Theorem 3.20 is not true.

**Example 3.21:** Consider the real line  $\mathbb{R}$  with usual the topology. Let  $I = \{\Phi\}$ . Then,  $\mathbb{R}$  is regular and hence  $\alpha$ Ig-regular. But open sets are not closed and hence open sets are not  $*$ -closed.

#### 4. Mildly $\alpha$ Ig-Normal Spaces

We introduce and study a new class of spaces called mildly  $\alpha$ Ig-normal spaces. The relationships between mildly  $\alpha$ Ig-normal spaces and new ideal topological functions are investigated. Moreover, we obtain characterizations of mildly  $\alpha$ Ig-normal spaces, and properties of new ideal topological functions for mildly  $\alpha$ Ig-normal spaces in ideal topological spaces.

**Definition 4.1:** An ideal topological space  $(X, \tau, I)$  is called mildly  $\alpha$ Ig-normal if for every pair of disjoint  $H, K \in RC(X)$ , there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $H \subseteq U$  and  $K \subseteq V$ .

**Theorem 4.2:** The following are equivalent for an ideal topological space  $(X, \tau, I)$ :

- (i)  $X$  is mildly  $\alpha$ Ig-normal.
- (ii) For any disjoint  $H, K \in RC(X)$ , there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $H \subseteq U$  and  $K \subseteq V$ .
- (iii) For any disjoint  $H, K \in RC(X)$ , there exist disjoint Ig-open sets  $U$  and  $V$  such that  $H \subseteq U$  and  $K \subseteq V$ .
- (iv) For any  $H \in RC(X)$  and any  $V \in RO(X)$  containing  $H$ , there exists an Ig-open set  $U$  of  $X$  such that  $H \subseteq U \subseteq cl^*(U) \subseteq V$ .
- (v) For any  $H \in RC(X)$  and any  $V \in RO(X)$  containing  $H$ , there exists an  $\alpha$ Ig-open set  $U$  of  $X$  such that  $H \subseteq U \subseteq cl^*(U) \subseteq V$ .

**Proof:** (i)  $\Rightarrow$  (ii): Let  $H, K \in RC(X)$ , where  $H$  and  $K$  are disjoint. Since,  $X$  is mildly  $\alpha$ Ig-normal, there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $H \subseteq U$  and  $K \subseteq V$ . Hence (ii).

(ii)  $\Rightarrow$  (iii): Let  $H, K \in RC(X)$ , where  $H$  and  $K$  are disjoint. By (ii), there exist disjoint  $\alpha$ Ig-open sets  $U$  and  $V$  such that  $H \subseteq U$  and  $K \subseteq V$ . Since every  $\alpha$ Ig-open set is Ig-open set, such that  $H \subseteq U$  and  $K \subseteq V$  where  $U$  and  $V$  are Ig-open sets. Hence (iii).

(iii)  $\Rightarrow$  (iv): Let  $H \in RC(X)$  and any  $V \in RO(X)$ . By (iii), there exist disjoint Ig-open sets  $U$  and  $V$  such that  $H \subseteq U$  and  $X - V \subseteq W$ . Hence  $X - V \subseteq int^*(W) \Rightarrow X - int^*(W) \subseteq V$ . Since  $U \cap W = \Phi$ , we have  $U \cap int^*(W) = \Phi$  and so  $cl^*(U) \subseteq X - int^*(W)$ . Therefore, we obtain  $H \subseteq U \subseteq cl^*(U) \subseteq V$  where  $U$  is Ig-open.

(iv)  $\Rightarrow$  (v): Let  $H$  and  $K$  be disjoint regular closed sets of  $X$ . Then,  $H \subseteq X - K$  where  $X - K \in RO(X)$ . By (iv), there exists a Ig-open set  $G$  of  $X$  such that  $H \subseteq G \subseteq cl^*(G) \subseteq X$

– K. Hence  $H \subseteq \text{int}^*(G)$ . If  $U = \text{int}^*(G)$ ,  $U$  is  $\alpha\text{Ig}$ -open set such that  $H \subseteq U \subseteq \text{cl}^*(U) \subseteq X - K \subseteq V$ .

(v)  $\Rightarrow$  (i): Let  $H$  and  $K$  be disjoint regular closed sets of  $X$ . Then,  $H \subseteq X - K$  where  $X - K \in \text{RO}(X)$ . By (v), there exists an  $\alpha\text{Ig}$ -open set  $U$  of  $X$  such that  $H \subseteq U \subseteq \text{cl}^*(U) \subseteq X - K$ . If  $V = X - \text{cl}^*(U)$ , then  $U$  and  $V$  are disjoint  $\alpha\text{Ig}$ -open sets of  $X$  such that  $H \subseteq U$  and  $K \subseteq V$ .

**Theorem 4.3:** Every  $\alpha\text{Ig}$ -normal space is mildly  $\alpha\text{Ig}$ -normal.

**Proof:** Let  $(X, \tau, I)$  be an  $\alpha\text{Ig}$ -normal space and  $A$  and  $B$  be any two disjoint regular closed sets in  $X$ . Since  $A$  and  $B$  are regular closed in  $X$ , they are closed in  $X$ .  $(X, \tau, I)$  is  $\alpha\text{Ig}$ -normal implies that there exist disjoint  $\alpha\text{Ig}$ -open sets  $U$  and  $W$  such that  $A \subseteq U$  and  $B \subseteq W$ . Hence  $(X, \tau, I)$  is mildly  $\alpha\text{Ig}$ -normal space

**Remark 4.4:** The following example shows that mildly  $\alpha\text{Ig}$ -normal space is not an  $\alpha\text{Ig}$ -normal space.

**Example 4.5:** Let  $(X, \tau, I)$  be an ideal topological space such that  $X = \{a, b, c\}$ ,  $\tau = \{\Phi, \{c\}, \{a, c\}, \{b, c\}, X\}$  and  $I = \{\Phi\}$ . Then  $(X, \tau, I)$  is a mildly  $\alpha\text{Ig}$ -normal space. Now,  $A = \{a\}$  and  $B = \{b\}$  are disjoint closed sets, but they are not separated by disjoint  $\alpha\text{Ig}$ -open sets. So  $(X, \tau, I)$  is not  $\alpha\text{Ig}$ -normal space.

**Theorem 4.6:** Every mildly normal space is mildly  $\alpha\text{Ig}$ -normal space.

**Proof:** Let  $(X, \tau, I)$  be an ideal topological space. Suppose that  $A$  and  $B$  are disjoint regular closed sets in  $X$ . Since  $X$  is mildly normal space, there exist open sets  $U$  and  $V$  such that  $A \subseteq U$  and  $B \subseteq V$ . But, every open set is  $\alpha\text{Ig}$ -open. Therefore,  $U$  and  $V$  are  $\alpha\text{Ig}$ -open sets such that  $A \subseteq U$  and  $B \subseteq V$ . Hence,  $X$  is mildly  $\alpha\text{Ig}$ -normal space.

#### 4. Conclusion

In this paper, a new class of spaces  $\alpha\text{Ig}$ -normal space,  $\alpha\text{Ig}^*$ -normal space, mildly  $\alpha\text{Ig}$ -normal space, are defined and their properties are discussed.

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